THE MATHEMATICS TEACHER

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ROUND TABLE ON THE TEACHING OF CALCULUS.

ELIZABETH B. COWLEY, Vassar College.—A leader of a round-table discussion is not expected to read a formal paper on a minute study of some one part of the topic, but rather to suggest those phases of the subject that will make his talk provocative of discussion. Or, to use a word recently coined, his talk should be "discussative."

As Mr. Smith suggested that it might be well for me to consider the teaching of calculus to girls, let us take that for the first topic. In order to get as quickly as possible to the vital part of the question, suppose we ask: How should a beginning course in the calculus intended for the average young woman in college differ from the course in that subject planned for the average young man in college? If you will permit me to take advantage of my position by replying to this question myself before giving anyone else a chance to do so, I shall answer most emphatically that there should be no difference. It is true that it is possible to find certain problems that might appeal more strongly to some boys than to some girls. But it is possible to find as great differences between different groups of boys. Take, for example, a set of boys whose previous training has all been definitely directed toward preparation for engineering work. Certain problems in calculus that have a very real and vital interest for them will be meaningless to a group of lads in a small rural high school in an agricultural community. Such differences in applications are, after all, superficial. There is a body of fundamental facts and a characteristic mode of

thinking that must be mastered by all. For success in teaching calculus, or any other subject, it is essential that the emphasis be properly placed. Recently we have all heard and read much cheap and superficial talk about the differences between girls' and boys' work in arithmetic and algebra. These speakers and writers seem to be so overwhelmed by the superficial fact that girls are not boys that they have lost all sense of perspective and have forgotten the important and fundamental fact that girls, as well as boys, are human beings. They are suffering from the same complaint that affected a physicist whom a foreign professor was once trying to describe to an American audience. After telling of the power and insight and industry of this man and how, nevertheless, he had never quite arrived at the goal, the professor said "The trouble with him was that he always got his empha'sis in the wrong places."

Let us turn, now, to some other questions that may prove of interest. When should a first course in calculus be introduced? There has been a tendency in colleges in recent times to push the elementary calculus back into the sophomore or freshman year. This is due, usually, to pressure from the allied departments, which need the elements of differentiation and integration by the end of the sophomore year, at least. Another question which this one suggests is whether it is possible to have any calculus in the secondary schools. If so, shall it be a formal course in calculus, or merely a placing of emphasis in the algebra upon the ideas of function, graph, etc.? The question of calculus in the secondary schools has received much attention in Europe.

Another point toward which discussion might profitably be directed is the relative emphasis to be placed, in a beginning course, upon the purely formal side and the applications.

Again, there may be considerable difference of opinion as regards the best time for introducing integration. For instance, is it wise to take up the simplest algebraic integrals before the study of the differentiation of the trigonometric functions?

The last topic which I shall suggest is the importance of laying sufficient emphasis upon the simplest underlying principles of the calculus. Too often students who can readily differentiate and integrate quite complicated forms have the humiliat-

ing (but stimulating) experience outside the class-room of being asked what the calculus really is and being unable to give any coherent and satisfactory answer. Of course, this is a deep question for a young student. But, is it not possible, even in a first course in the calculus, for the student to gain a clear idea of the simplest fundamental principles? By so doing, he will greatly increase the pleasure and profit to be derived from his work.

W. H. METZLER, Syracuse University.—In considering the teaching of calculus to students in a college of liberal arts this speaker emphasized the importance of students having a very thorough knowledge of the derivatives and integrals of all fundamental forms. Integration is an art as well as a science and students should be taught the short and concise way of integrating any given expression. To this end it is poor training to use, as illustrations of any given method of integration, examples which would better be worked by some other methods. Students should not use a table of integrals but should learn how to integrate the type forms.

WILLIAM J. BERRY, Brooklyn Polytechnic Institute.—My colleagues, I know, will excuse me if I confine my remarks about the teaching of calculus entirely to the question of the instruction of engineering students. All my professional life has been spent in engineering schools, and I prefer to speak only of those things with which I am acquainted. There are two problems to the consideration of which I invite your attention for a few moments. First, what shall be the attitude of the teacher of calculus toward his subject? and second, what is the general attitude of the student toward the calculus?

In an article published in the *Engineering Record* for February 27, 1915, Professor Swain, of Harvard University, says: "I think that the fact that engineering is to so large an extent a mathematical subject is one of the main reasons why the engineer is not recognized. In my opinion, there is scarcely anything that tends more to narrowness of view than dealing all the time with problems that can be solved only by rigid mathematical processes, because the problems which confront us are

not mathematical, the data are shifting, variable, and the human element comes in."

Within recent years there has arisen what is sometimes referred to as the Perry method of teaching Calculus, and in my opinion there may be some connection between this method and the attitude which it represents, and the criticism which I have just quoted. I am quite sure that all of us, as teachers of mathematics, dissent most strongly from the point of view expressed by Professor Swain. It is gratifying to find that he does not voice the sentiments of all the members of his profession. In the *Record* for March 20, 1915, Mr. Allen Hazen, commenting upon Professor Swain's article, says: "To the writer one of the most surprising things in Professor Swain's statement is his reference to the narrowing influence of mathematics. The writer has been inclined to think that the study of mathematics is useful in tending to clear and logical thinking and accurate expression, and he has been disposed to think that the engineering schools might well require more mathematics rather than less. The fact that the practical problems of life are not mathematical ones, but are very much more complex, is another matter, and one that the man must adjust himself to when he encounters the problem of life."

And a little later Mr. L. C. Fritch says: "There is no better training for the mind than a course in mathematics, and in this respect the training of the engineer is superior to that of the lawyer or business man. Mathematics is more truly an exact science than the law or any other of the arts."

Although we do not agree with Professor Swain, yet his criticism, coming from so eminent an authority, must necessarily give us pause, and we should inquire carefully into the subject and endeavor to find out to what extent such strictures are justified. The teacher may take either of two attitudes with regard to the presentation of the calculus to students of engineering. He may consider it as a tool, of which they must become the masters, which they must be able to use with facility and accuracy, and he may rest content with that. Or he may regard it as a science in which they are to be instructed, which is not only to teach them facts, but to develop their power of thought, to stimulate their imagination, and to give them the

vision without which neither mathematician nor engineer can be truly great. He is concerned rather with the fundamental principles and methods than with applications; his students are to be thinkers and not merely differentiating or integrating machines. The Perry method takes rather the first of these two points of view, and it is my opinion that taught in this fashion the calculus may well be occupying more of the time of the engineering student than is justified, and may lead to precisely that type of narrowness of which Professor Swain speaks. If, on the other hand, the teacher regards his work as the interpretation of one of the great branches of mathematical science, if he feels that his mission is not to cram the mind of his student with facts and formulas, to prepare them to compute radii of gyration and moments of inertia for engineering handbooks, but to stimulate their imaginations, to clarify their ideas, and to give them power of thought and accuracy of expression,—then I believe that the time allotted to the calculus is rather too short than too long.

When we come to a consideration of just what topics should be included in a first course, what methods of proof should be used, how far mathematical rigor should be insisted upon, we find that the questions which present themselves are not at all easy to answer. The most satisfactory course that has yet been outlined is probably the one prepared by the committee of the Society for the Promotion of Engineering Education and published in its syllabus. Even then the teacher will find that there is more than can be adequately presented in a course of a hundred hours. Personally, I believe that the time allotted to the calculus in a school of engineering should be about fifty per cent. more than this.

Mathematics is a science which above all others should be honest and straightforward. I think it a grave mistake to present to the student as a proof an argument which we ourselves know to have weak points in it. You may say, and rightly, that it is not possible to give the student in his second year in college rigorous proofs of the various theorems needed in the calculus. That I cheerfully grant, but I think in fairness to him, and as a matter of common honesty, we should point out the deficiencies, assure him that the rigor that is lacking can

be supplied, and explain to him that the reason that it cannot be done then and there is because of his own inadequate preparation and insufficient background. My personal experience has been that the student does not doubt the truth of the theorem when you tell him that what you present is not a proof, but is merely making probable the theorem through geometric intuition; and I know, too, that if in more advanced work he finds that the demonstrations that have been given him are faulty, he begins to suspect not only all the demonstrations with which he is familiar, but the accuracy and truth of the theorems as well.

I believe that the justification for the inclusion in an engineering course of a course in the calculus is due not so much to the fact that it will fit the student more quickly for the earning of a living wage, as to the far more important fact that it is the basis of all future work in analysis, that it lies at the foundation of many of the most important branches of engineering, that progress in the development of the science (and this is particularly true of electrical engineering) is conditioned largely by familiarity with mathematical processes and facility in their use. As an example of this, I need only to cite the very remarkable work of Professor Kennelley, of Harvard University, in the application of functions of complex variables to problems of alternating-current electricity.

And now what about the attitude of the student? We are all quite familiar, I am sure, with the fact that the calculus is usually the best hated course in the institution, that in more than one school those who have successfully passed their final examination in the subject join in a solemn *auto da fe* in which the offending text-books are formally committed to the flames. Personally, I never have been able to see why this should be so. I do not believe that there are any great difficulties inherent in the subject, and am constrained to the opinion that the trouble is either with the method of presentation or with the inadequate preparation of the students themselves. I should be very loath to believe that the former explanation is the true one, for the attitude of the student seems to be pretty much the same even when the teachers are men of whose ability as mathematicians and as instructors, we can have very little doubt.

For some years I have been studying this subject very care-

fully in my own classes and have found that in perhaps nine cases out of ten when a student failed to understand a demonstration it was because the algebra, the trigonometry, or the analytic geometry employed had exceeded his knowledge of the subjects; that when a student failed to get a problem the trouble was more apt to be with the manipulation of the quantities involved than it was with the fundamental principle of the calculus underlying it. The new ideas introduced into the calculus are comparatively few, and with careful explanation ought to give no unusual amount of trouble. If we examine the average text-book we shall find that it consists very largely of applications of these few principles. In my own institution we have succeeded to a very satisfactory degree in eliminating much of this feeling against the calculus, and it has been accomplished by calling the attention of the students, publicly and privately, not in a general way but very specifically, to the fact that their difficulties are due, not to the subject that they are at present studying, but to the subjects with which they had fondly supposed they were finished and which they thought were forever put behind them.

Let me say in conclusion that if the teacher of the calculus in an engineering school will steadfastly refuse to lower his ideals, to turn what is a wonderful science into a mere tool, if he will endeavor to turn out men who have power to think and to analyze rather than mere perambulating integraphs, if he will remind the student wherever possible that the trouble is not with the calculus but with himself and his lack of preparation, he will find not only that the results are more satisfactory, but that the work itself will bring that interest and that satisfaction which are the teacher's true reward.

CHARLES C. GROVE, Columbia University.—It was of interest to hear Professor Metzler's experience and practice with regard to tables of integrals. I have not ceased to rejoice that I was not brought up to use tables as a student. I feel that if the tables are introduced before the student has mastered the casting of integrals into standard forms and integrating them by observation, before he has learned to use rather freely the special method of integration, and the special transformations

that make integrals readily integrable, he comes to rely on the tables and will likely never become proficient in the art of integrating.

Where a full year is devoted to the first course in the calculus, I prefer to make no mention of tables during the first half-year. For example, I would cover the first 200 pages of Davis or 133 pages of Osgood before giving tables and would and do give more methods and substitutions of integration than given by either of these authors.

Further, many integrands are not given in a form that permits the use of tables directly. The student needs to learn how to recast these integrands and by the time he has done that the integral is very often in a form that may be integrated by observation, if he has learned twenty to twenty-five standard forms through using them frequently. For example, of the 164 integrals given in Osgood, pp. 446–451, almost all are more easily and quickly integrated without reference to Peirce's Short Table of Integrals than by trying to cast them into a form to which the table may be applied.

The privilege of using tables early may readily become a snare and a delusion. Later, after the student has learned how to integrate without resort to tables, for rapid work in practice the tables may well be used and have their proper place.

MATHEMATICS CLUBS IN THE HIGH SCHOOL.

By C. A. Snell.

It is barely possible that some of you have so many high-school clubs and societies, with the effects of which you have to contend, that the suggestion of another club may not meet your approval. The kind of club, however, that I have to describe to you is based upon scholarship, proficiency and interest in mathematics and, from its very nature, would not attract the element that enjoys "adding tone" to societies.

As a matter of fact, if the teacher has time enough to spare from the required work in the mathematics courses, the club work could be carried on in the class room. But, in many high schools where a large per cent. of pupils are fitting for college, and in special fitting schools there is little time for other than the required work for admission to colleges.

In presenting this subject of mathematics clubs in the high school, I am, with your permission, going to refer to the work of our mathematics club in Melrose, which we formed a year and a half ago. We felt that an organization of boys and girls who had manifested ability and interest in mathematics would stimulate a desire for further development individually, and would furnish an opportunity for expression of individual ideas and opinions, and for applications of theory. Furthermore it was thought that a certain standard of attainment, required for admission to the club, might be set up for beginners in high-school mathematics.

That the latter has had effect was shown last fall when a boy applied for admission to the club. On looking up his record it was found that the average attainment in his first two years' work was not high enough to admit him. "Well," said he, "I could have done a great deal better if I had known this club was to be organized."

It does not always follow that a certain marking is adhered to. If a pupil has shown the right attitude toward his work in mathematics and has developed a certain skill and ingenuity

for construction, and a certain amount of executive ability, his admission is ruled on by the society. I believe if a boy has anything to offer or to receive from a club of this kind he should not be deprived of the opportunity to express his ideas, or to develop along a line in which he has some natural aptitude.

The charter members of our club were juniors and seniors who had elected mathematics and had attained an average therein of 85 per cent. That standard for admission they still maintain, but exceptions, as I previously stated, are ruled upon. This year they have admitted members from the sophomore, or second-year class, whose standard up to April 1 would warrant their admission.

After becoming a member of the club, if a pupil drops below the average of 80 per cent. in his mathematics, he is given a month's notice to bring up his work or he will be dropped from membership. The chairman of the membership committee interviews the teachers as to the standards of the members, and thereupon notifies anyone who may be neglecting to keep up the standard set. The mathematics teachers and the principal are associate members and are always welcome to any club meetings.

The officers are the president, a boy chosen from the senior class, a first and a second vice-president, secretary and treasurer, and three executive committees. They have drawn up a constitution which states that the first vice-president shall be chairman of the *student aid committee,* and the second vice-president shall be chairman of the *program committee;* the third executive committee is the membership committee, already mentioned.

Their meetings are held once a month, and, after the business, is in the hands of the program committee. The club numbers about forty active members, both boys and girls. There was some difference of opinion at first as to whether or not girls should be admitted to the club, but decision was finally made in their favor, and up to the present time no girl has been dropped from membership on account of deficiency in mathematics.

Their programs consist of subjects chosen with reference to their mathematical or scientific interest. Various members are

particularly interested in one phase or another and they are always willing to present their favorite topics. They have a small mathematics library and they also draw material from current magazines. The mathematics teacher occasionally furnishes a topic for their programs. Considerable time is spent in mathematical recreations: one boy has just constructed a small adding machine; a girl wrote a most creditable play in blank verse, which fifteen of the boys presented at their mid-year reunion. A certain portion of time is devoted to considering mathematical subjects not offered in the school curriculum. Occasionally a speaker on some mathematical or scientific subject meets with the club. The members have greatly enjoyed Professor Ransom and Mr. Power, of Tufts, who have kindly presented subjects of unusual interest.

They have designed a pin which consists of a circle inscribed in an equilateral triangle. Within the circle is the figure of the Pythagorean theorem and the Greek word "Ευκλείδης" for Euclidean, the name of their club.

Perhaps the most essential feature of the club, to the teacher at any rate, is the work of the student aid committee. This is a committee made up of twelve or fifteen members of the club, who are appointed to assist the teachers of the mathematics department. On such occasions backward pupils can be given more careful assistance and those who wish to make up work lost by absence may receive individual attention.

Again, some of the most proficient members of the club have been entrusted to correct papers for the teacher, and in one or two instances have taken classes in the absence of a teacher. Two or three have tutored beginners, and last year one of the senior girls tutored a classmate, with the result that the latter passed her entrance examination in algebra for Simmons.

In consideration of this talk, I asked for an expression of opinion from the members present at the last meeting of the club on the question: "What advantage has the mathematics society been to you and in what particular phase of the work have you been interested?"

The president of the senior class said:

"The mathematics club has helped me in three distinct ways: First, it has given me a knowledge of the less common geo-

metrical and algebraic problems and the easier methods of working out those problems. Second, it has brought me into contact with other fellows interested in scientific and mathematical work, and third, it has enabled me to know of new inventions by hearing pupils give talks about them; therefore, I am better able to understand lectures given on science and mathematics and to comprehend articles written on these subjects."

Another wrote: "I have become more interested in geometry than ever before."

A third said: "The mathematics society has given me something to study for."

A fourth said: "I enjoy the freak geometry theorems."

On asking different members of the student aid committee what benefit they have derived from helping others, I received the following replies:

"I enjoy what I have done on the student aid work: it has helped me to review my work and has given me a real desire to teach geometry."

Another said: "New ideas are brought to my mind and I find wherein I am weak in mathematics."

Still another wrote: "Working on the student aid committee has taught me to be more accurate in my own work."

From those who have been helped by the committee I have received the following: "I think I sometimes receive more help from the students than from the teacher, for I feel freer to ask as many questions as I like, whereas I would feel diffident in asking a teacher."

A second stated: "I have never been helped by the teacher so I cannot compare the teacher with the student helper, but I can say that the one who helped me cleared up everything I did not understand."

A third: "I think a student can help another in a great many cases where perhaps a teacher cannot. Often the teachers have gone over the different parts of the subject so much that they do not realize the difficulty of a beginner, but a student who has just mastered the points can realize better where the difficulty lies."

One boy expressed himself: "A fellow near your own size and age doesn't think you are so much of a lunk head when you ask all the questions you want to."

One reply showed quite another phase: "They didn't help me much because I wouldn't let them. I wanted to help myself."

Several very loyally expressed themselves to the effect that no pupil's help could equal that of the teacher's, and one quite correctly explained his statement by adding: "The helper *tells what he* knows, while the *teacher teaches* the pupil to know for himself."

These illustrate a few instances where, I think, a student can, under the supervision of the teacher, be of great assistance, and, at the same time, strengthen his own grasp of the subject.

The advantage to a teacher of a trustworthy helper is obvious, especially if the teacher has large or poorly graded classes.

Of course there are certain types of pupils we cannot turn over to student helpers; nevertheless, I am sure that our mathematics teachers agree that the club is very much worth while, not only as an incentive to beginners, but as a most valuable training for those who are active members of the club.

As it was stated in the beginning, a considerable portion of the work done by the club can be carried on in connection with class-room work if time permits. I think it is a general experience of teachers that some degree of recreation is productive of more interest in the subject at hand, and, hence, of better results.

There is a large field for work and creative interest—history and contributions to mathematics and science, debates, an innumerable list of recreations, tricks, puzzles, fallacies, curiosities and constructions. Even if certain questions, technically presented, are beyond their comprehension, their interest and imagination are aroused, their view of things is broadened, and they cannot help but be impressed with the universality of mathematics.

At our last meeting of the year I have proposed that we trace, in an elementary way of course, the laws of mathematics as applied to other high school subjects; that we consider their bearing upon the languages, history, art, etc.

A phase of the club work worth consideration is the offering of prizes for the best examinations passed by beginners of algebra and of geometry, or to the pupil who has made the best progress in his year's work.

To any teachers present who would be interested in "The Pedagogic Value of Recreations," supplemented by a most interesting bibliography, I would suggest an article in the April number of *School Science and Mathematics* on "Recreations in Secondary Mathematics," by Chas. W. Newhall.

The article expresses in a more general and a broader way than I have done, the advantages of stepping aside from the regular routine, even in a most elementary way. If extra time is required of the teacher in forming clubs of the kind I have described, the effort is duly rewarded in the added interest that pupils manifest in their daily tasks.

Whatever the method used in teaching may be, the thing to be desired is a vital interest in the subject studied, and I most heartily recommend the mathematics club as one method of arousing such an interest, and of increasing personal and classroom efficiency.

HIGH SCHOOL,
MELROSE, MASS.

THE STUDY OF MATHEMATICS AS AN AID TO GENERAL LOGICAL POWER.

By Leonard M. Passano.

In a paper read before a joint meeting of the New England Association of Mathematics Teachers and the Association of Teachers of Mathematics of the Middle States and Maryland,* Mr. George G. Chambers analyzes the answers to a set of simple questions in logical reasoning made by forty-four "high school girls who had just completed a half year's work in plane geometry, . . . the first two books," and who had previously "studied algebra through simultaneous quadratics, spending on that five periods a week for one school year and two periods a week of one fourth of a school year, . . . followed by two periods a week for one fourth of a school year in constructive geometry." Before reading his paper Mr. Chambers obtained answers to the same set of questions from forty-eight persons present at the meeting.

It seemed to the present writer that a similar test made on other classes of students might prove of interest for purposes of comparison. It was also thought that some light might be thrown on the question whether the study of mathematical subjects strengthened the general "logical faculty." The writer, therefore, obtained answers to the set of questions from thirty-two freshmen and thirty-one sophomores of the Massachusetts Institute of Technology, at a time when the students had nearly completed the current year's work. Also, through the courtesy of Professor Sarah E. Smith, answers to the questions were obtained from twenty sophomore and twenty-one junior students of Mount Holyoke College, at about the time of completion of the year's work.† These respective groups had had, ap-

*"A Study of the Reliability of Test Questions." See the Mathematics Teacher for March, 1915.

†The mathematics of Institute freshmen includes plane and solid analytic geometry and differential calculus; that of the sophomores, integral calculus and very elementary differential equations. The Mt. Holyoke sophomores had "had a half year of analytic geometry and a half year of calculus"; the juniors, "one and a half years of calculus."

proximately, the same amount of mathematical training. The technology students belonged to non-mathematical courses, and were by no means marked by exceptional mathematical ability; rather the reverse. One would suspect that the Mt. Holyoke students in question are a group of rather select individuals.

In the case of all classes of persons experimented upon, to quote Mr. Chambers, "particular care was taken to see that there was nothing said or done to indicate in the minds of the pupils that there was any connection between this test and their work in mathematics. No inquiries in regard to the meaning of the questions were answered." In the case of the Institute students the test was unannounced and unexpected, and Professor Smith gives the same assurance in respect to the Mt. Holyoke students. Mr. Chambers does not state how much time was allowed to his experimentees. With one or two exceptions, twenty to twenty-five minutes were found to be sufficient time for the college students, and in marking no distinction was made between those who returned the answers quickly and those who made use of the entire period of about fifty minutes.

The questions asked were the following:

I. Do you discover any defects in the following reasoning, and if so, explain why it is defective.

The sidewalk was wet this morning. Therefore it must have rained last night.

II. If all the inhabitants of the Rahib Islands have blue tattoo marks on their bodies, then which of the following statements would necessarily be true, which could not be true, and which might possibly be true?

1. All people who have blue tattoo marks on their bodies are inhabitants of the Rahib Islands.

2. Some inhabitants of the Rahib Islands do not have blue tattoo marks on their bodies.

3. No people with blue tattoo marks on their bodies live anywhere except on the Rahib Islands.

4. Some of the inhabitants of the Rahib Islands have blue tattoo marks on their bodies.

III. A certain club wishes to select the evening for its regular weekly meeting which would be most satisfactory to its members. Accordingly the secretary wrote to each member, asking what evening would be most satisfactory.

Can you suggest another question which would have been better for the secretary to have asked?

IV. If a photographic plate be exposed to X-rays, and then developed, black marks will be found upon it.

1. If upon developing a photographic plate you should find black marks upon it what would you conclude?

2. Also if you should not find black marks upon it what would you conclude?

V. If John agrees to join the football team provided Charles joins it, but Charles decides not to join it, what follows about John? If John joins, but Charles does not join, is John breaking his agreement?

The writer will not enter into any defence or criticism of the above questions; in marking them he followed, as exactly as possible, the plan outlined in Mr. Chambers's paper, to which reference has been made.*

In the following tables and plates the high-school girls are represented by the letter S, the group of teachers by T, Institute freshmen by I_1, Institute sophomores by I_2, Mt. Holyoke sophomores by H_1, Mt. Holyoke juniors by H_2.

TABLE I.

Points Obtainable.	Number Obtaining.						Percentage Obtaining.					
	S.	T.	H_1.	H_2.	I_1.	I_2.	S.	T.	H_1.	H_2.	I_1.	I_2.
9		18	3	7	6	7		38	15	33	18	22
8	1	12	7	7	10	11	2	25	35	33	30	34
7	3	7	6	4	3	6	7	15	30	19	9	19
6	11	6	3	1	8	3	25	13	15	5	24	9
5	11	2		1	4	3	25	4		5	12	9
4	5	3	1		1	2	11	6	5		3	6
3	10						23					
2	2			1	1		4			5	3	
1	1						2					

TABLE II.

Points.	Percentage Obtaining.					
	S.	T.	H_1.	H_2.	I_1.	I_2.
7 or over.....	9	78	80	85	57	75
8 or over.....	2	63	50	66	48	56

An examination of the graphs seems to furnish some interesting information. The high-school girls (S) and the teachers (T), Fig. 1, give such results as one might expect; the former an approximate probability curve and the latter a curve begin-

* The present writer marked the Mt. Holyoke as well as the Institute papers, so that the same "personal equation" appears throughout.

ning farther to the right and rising to a maximum at nine points. The writer confesses to some surprise that no more than 38 per cent. of the teachers answered all nine questions correctly. The

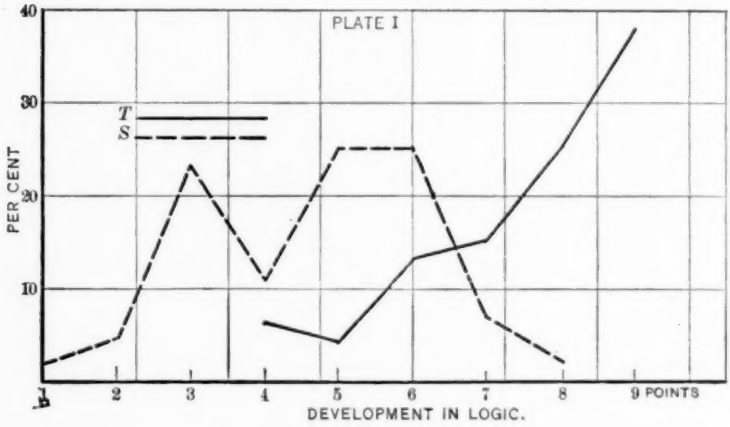

FIG. 1. Development in Logic.

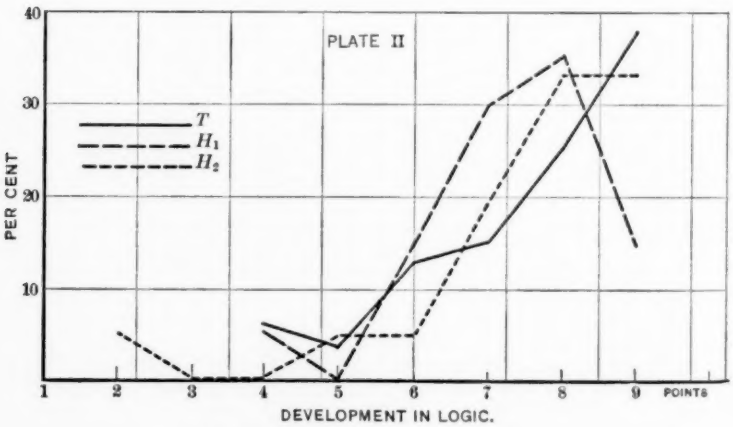

FIG. 2. Development in Logic.

teachers' curve may be looked upon as a standard; it is repeated, for purposes of comparison, in Figs. 2 and 3. That the graphs begin farther to the right as one passes to the higher grades is due, doubtless, to the weeding out of weaker students.

Fig. 2 compares the sophomore (H_1) and junior (H_2) students of Mt. Holyoke College, and shows, on the whole, an advance with longer training. Fig. 3 makes the same comparison, with like conclusions, for the two groups of Technology

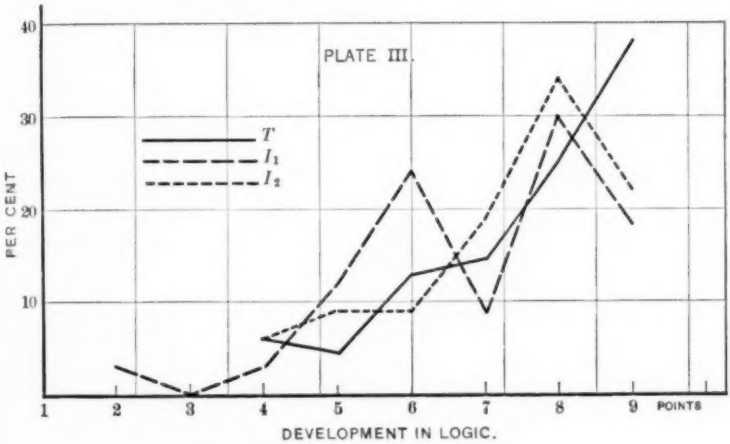

Fig. 3. Development in Logic.

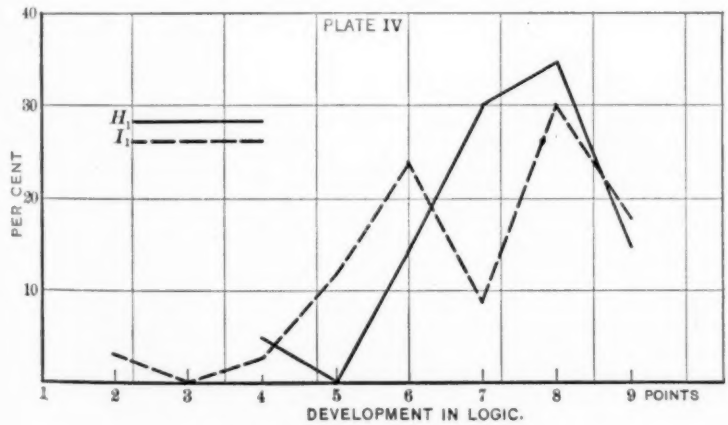

Fig. 4. Development in Logic.

students (I_1 and I_2). Figs. 4 and 5 make respectively a comparison between lower classmen (H_1 and I_1) and upper classmen (H_2 and I_2) of the two colleges.

As a further test, after an interval of about three days, the four groups of college students were asked to write the *questions* which had been asked in the preceding test. It was hoped

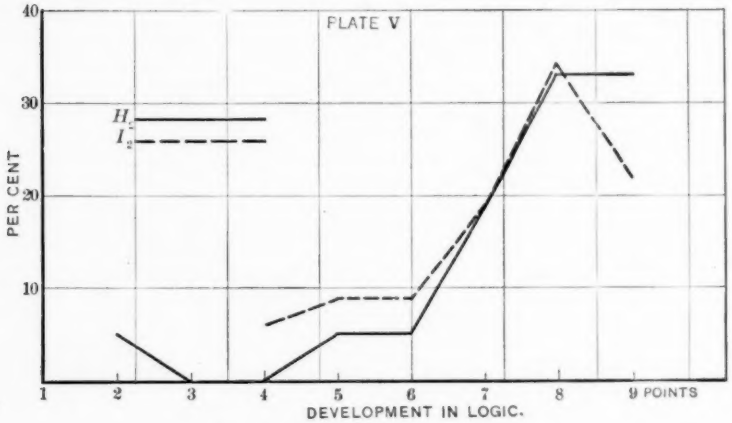

FIG. 5. Development in Logic.

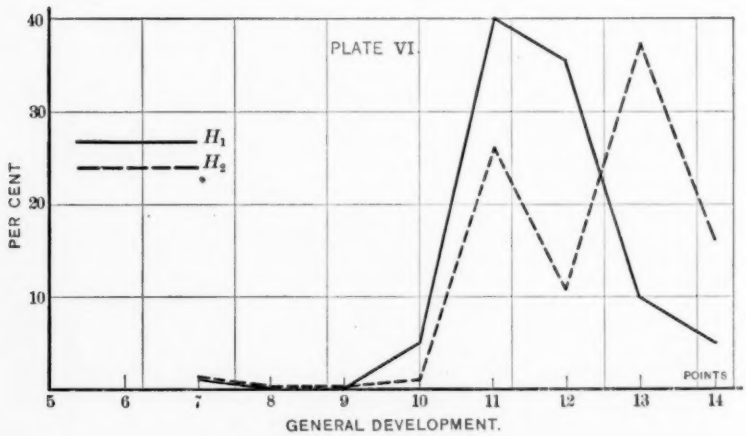

FIG. 6. General Development.

in this way to obtain data by which to evaluate the memory of the students for essentials. For this purpose each of the questions received one mark, making a total of five. In question II. a student remembering the primary statement and any one

of the sub-statements or conclusions received full credit for the question. Number IV. was similarly treated. There were thus fourteen points in all (9 logic; 5 memory) to measure total mental development—if it may be permitted to call it so. The

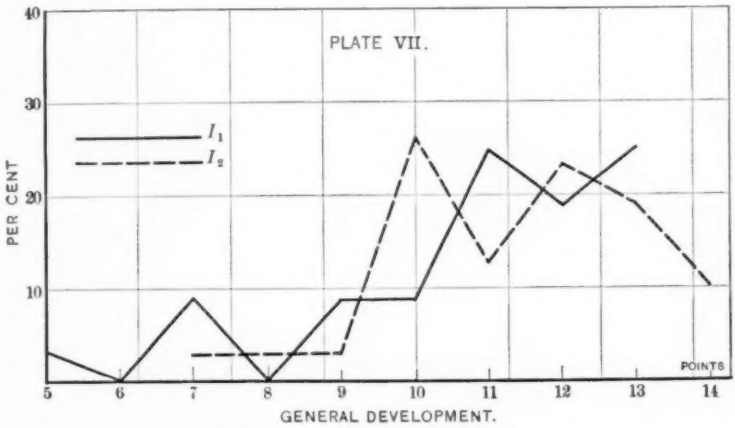

FIG. 7. General Development.

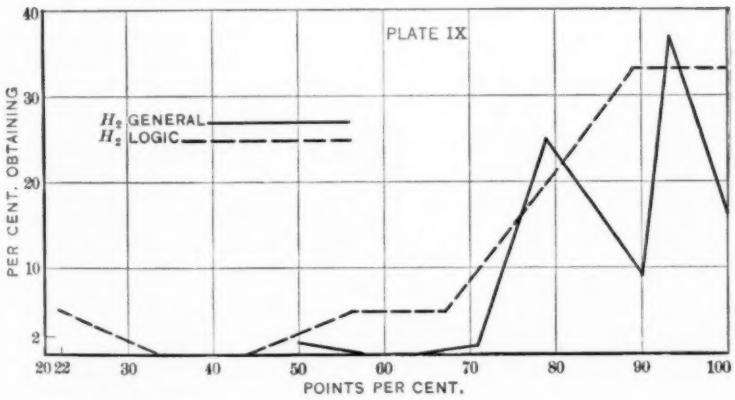

FIG. 8.

following tables, III. and IV., and figures 6 to 11 give the results obtained. Figs. 6 and 7 give the results of Table III graphically. Figs. 8, 9, 10 and 11 compare, for each group of students, the logical development with the general development, the horionztal axis giving percentages instead of points.

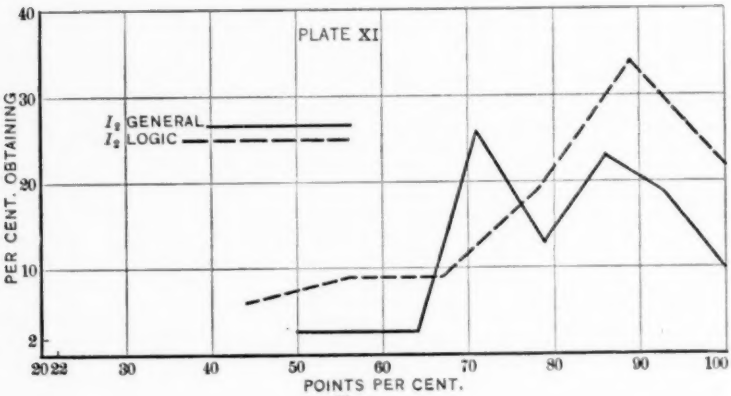

FIG. 11.

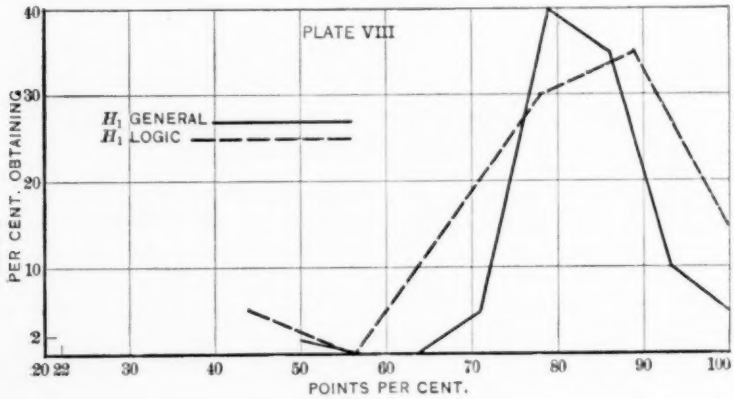

FIG. 9.

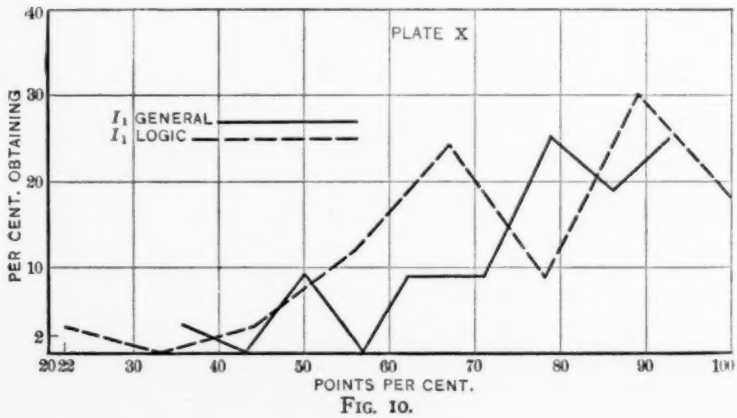

FIG. 10.

The writer would not too boldly draw conclusions from the few data of the above tests, but he believes that it may be asserted that evidence is here presented that training in mathematical studies does to some degree train the general logical faculty as well as memory in general, and that the increase in

TABLE III.

Points.	Percentage Obtaining.			
	H_1.	H_2.	I_1.	I_2.
14	5	16		10
13	10	37	25	16
12	35	11	19	23
11	40	26	25	13
10	5	1	9	26
9			9	3
8				3
7	1	1	9	3
6				
5			3	

TABLE IV.

Points.	Percentage Obtaining.			
	H_1.	H_2.	I_1.	I_2.
12 or over.....	50	64	44	52
13 or over.....	15	53	25	29

logical development is greater than the increase in general development.

Mr. Chambers, in his paper, calls attention to two "unethical" high-school girls, one of whom, in reference to question V., said that John should not be considered as breaking his agreement "because the matter was not of such importance that to change his mind could be called breaking his agreement." The other said that John breaks his agreement "provided the agreement was written and signed by both parties"; otherwise John "is eligible to break his word." It may be remarked that two of the Institute of Technology freshmen held the latter unethical view, but none of the sophomores and none of the Mount Holyoke students. It is pleasing to draw the conclusion that both the technical school and the woman's college tend to the improvement of morals.

In conclusion it may be remarked, with reference to question III. in the memory test, that to all of the young men the club was simply a club. To one young woman it was a "man's" club; to two a "woman's" club; and to a fourth a "prominent" club. Finally, one young woman, intolerant of the slowness of the post, asserts that the secretary "called up" the different members of the club!

MASS. INSTITUTE OF TECHNOLOGY,
BOSTON, MASS.

INSCRIBING REGULAR PENTAGONS AND DECAGONS.

By Joseph Bowden.

The chief object of the following paper is to prove exact a construction for the regular pentagon that is found in books on mechanical drawing* and is on that account thought by some to be inexact.

Let OA (Fig. 1) be a radius of the circle.

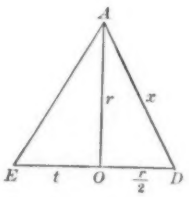

Fig. 1.

Draw OD perpendicular to OA and equal to half the radius. Draw DA and prolong DO to E so that $DE = DA$. Join AE.

Then EO and AE are respectively equal to the sides of the regular inscribed decagon and pentagon.

To prove this let $OA = r$, $DA = x$, $EO = t$.

Then $OD = r/2$. Hence $x^2 = r^2 + (r/2)^2 = 5r^2/4$.

Also $x = AD = ED = EO + OD = t + r/2$.

Therefore $5r^2/4 = x^2 = t^2 + rt + r^2/4$.

Hence

$$r^2 = t^2 + rt,$$
$$t^2 = r^2 - rt = r(r - t),$$
$$r/t = t/(r - t).$$

Now, in the triangle AOD, $x > r/2$.

Therefore, since $t = x - r/2$, t is positive.

* See Anson K. Cross, "Mechanical Drawing," Boston, Ginn & Co., p. 16; William F. Willard, "A Practical Course in Mechanical Drawing," Chicago, Popular Mechanics Co., p. 26.

Also, since $t^2 = r(r-t)$, $r-t$ is positive, and therefore $r > t$.

Therefore, since $r/t = t/(r-t)$, $t > r-t$ and t is equal to the greater segment of the radius when the latter is divided in extreme and mean ratio.

Therefore t is the side of the regular inscribed decagon.

Now

$$\frac{t}{r} = \frac{r-t}{t} = \frac{r}{t} - 1.$$

Hence

$$\frac{r}{t} - \frac{t}{r} = 1.$$

Squaring,

$$\frac{r^2}{t^2} - 2 + \frac{t^2}{r^2} = 1,$$

$$\frac{r^2}{t^2} + \frac{t^2}{r^2} = 3.$$

Hence

$$\frac{t^2}{r^2} = 3 - \frac{r^2}{t^2}.$$

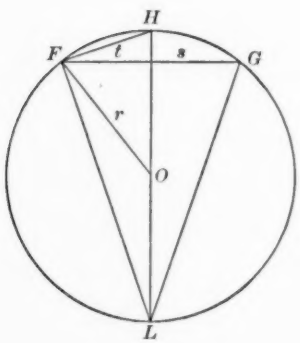

FIG. 2.

Now suppose that FG (Fig. 2) is a side of the regular inscribed pentagon and HL a diameter perpendicular to FG, cutting the smaller of the arcs subtended by FG in H. Let $FG = s$.

Then FH is a side of the regular inscribed decagon.

Now $FL^2 = HL^2 - FH^2 = (2r)^2 - t^2 = 4r^2 - t^2$.

Also, since the triangles FLG and FOH are similar,

$$FG : FH = FL : FO, \text{ or } s : t = FL : r.$$

Hence

$$\frac{s^2}{t^2} = \frac{FL^2}{r^2} = \frac{4r^2 - t^2}{r^2} = 4 - \frac{t^2}{r^2} = 4 - \left(3 - \frac{r^2}{t^2}\right) = 1 + \frac{r^2}{t^2}.$$

Therefore

$$s^2 = r^2 + t^2.$$

Now, in Fig. 1, $AE^2 = r^2 + t^2$. Hence $AE^2 = s^2$ and $AE = s$. Therefore AE is the side of the regular inscribed pentagon.

We see also, incidentally, that *if the radius and the side of the regular inscribed decagon are taken as the perpendicular sides of a right triangle, the hypotenuse will equal the side of the regular inscribed pentagon.**

The above proof does not involve radicals. The reader may be interested to work out one using radicals.

If the quadratic equation $r^2 = t^2 + rt$ is solved for t, we get, since t must be positive, $t = \frac{1}{2}r(\sqrt{5} - 1)$.

If this value is substituted for t in the equality $s^2 = r^2 + t^2$ and this equation solved for s, we get $s = \frac{1}{2}r\sqrt{10 - 2\sqrt{5}}$.

These formulas for t and s are well known.

ADELPHI COLLEGE,
BROOKLYN, NEW YORK.

* See Phillips and Fisher's "Elements of Geometry," New York, American Book Co., p. 476, prob. 125.

A PRACTICAL SCHEME FOR REVIEW OF IMPORTANT PROCESSES IN ARITHMETIC EIGHTH GRADE WORK.

By Laura Holbrook.

A. Expenses.

A'. Interior Improvements of School Building.

I. Have the children estimate the cost of tinting the side walls and ceiling of their class room. Measurements can be made by two pupils and the class can use their figures.

II. Estimate the cost of laying a new wood floor in the class room. One of the pupils can learn from a carpenter the price per square foot of laying Georgia pine.

B'. Exterior Improvements of School Building.

I. Estimate the price of resodding the front school yard, allowing for one or more circular flower beds of four foot diameter.

II. Estimate the cost of geraniums for bed at 10 cents a plant. At least fifteen plants would be needed.

B. Payment.

1. Draw checks on a local bank for payment of above expenses and make out in favor of the painter, the carpenter and the florist respectively.

C. Resulting Problems.

1. The carpenter deposits his check in bank leaving it to draw compound interest. Estimate approximately how much he will have to his credit at the end of fifteen years from date.

2. What per cent. of the entire expense was the bill of the painter?

3. If the florist made 25 per cent. by his bill, what was the cost of material and labor to him?

4. The painter loaned his income for the work, to a friend at

6 per cent. interest. His friend did not repay until the end of three years. What amount was then due him?

5. An agent charged the carpenter 2 per cent. commission for getting him the work. What was his commission?

6. Had the carpenter been a personal friend of any member of the school board he would have given them 5 per cent. discount on the bill. What would have been the amount the school board would have paid him in that case?

D. General Problems.

1. Let one of the pupils find out the amount voted for school purposes in the town. Also the value of the taxable property. What is the rate of taxation?

2. Find the value of the school property. What is the annual premium for insurance at 1½ per cent.?

Jersey City,
N. J.

ON THE USE OF THE TERM CANCEL.

By Elijah Swift.

All who have had occasion to correct work—particularly that done by poorer pupils—involving fractions must have frequently noticed cases where students "simplify" fractions by *subtracting* the same quantity from numerator and denominator. This is very noticeable in work on trigonometric identities, where students are apt to be careless with their algebra in their eagerness to "make the identity prove." For instance a harmless expression like $\frac{\sin x - \cos x}{\sin x + \cos x}$ becomes $\frac{\sin x}{\sin x}$ after "cancelling" $\cos x$. Is it possible that such processes as these are due to a confusion arising from the two uses of the word "*cancel*"?—for we use the word to denote *dividing* numerator and denominator, or both sides of an equation, *by the same factor,* as well as *subtracting the same quantity* from both sides of an equation. If it would result in clearer thinking, it might be well to reserve the term *cancel* for the process of *division by a common factor,* and to use the expression *subtract from both sides of the equation* for cancel in the other sense.

University of Vermont,
Burlington, Vt.

A STUDY OF THE RELIABILITY OF TEST QUESTIONS.

By C. E. Rogers.

This paper is in the nature of a supplement to "A Study of the Reliability of Test Questions," by Professor George Gailey Chambers, in the March number of The Mathematics Teacher. It includes results from five groups of students secured by the use of the same questions upon which Professor Chambers's study was based, except in the case of one group, where I added two questions to the original list for a purpose which I shall point out later.

For easy reference I insert the questions as originally prepared by Professor Chambers.

I. Do you discover any defects in the following reasoning, and if so, explain why it is defective.

The sidewalk was wet this morning. Therefore it must have rained last night.

II. If all the inhabitants of the Rahib Islands have blue tattoo marks on their bodies, then which of the following statements would necessarily be true, which could not be true, and which might possibly be true?

1. All people who have blue tattoo marks on their bodies are inhabitants of the Rahib Islands.

2. Some inhabitants of the Rahib Islands do not have blue tattoo marks on their bodies.

3. No people with blue tattoo marks on their bodies live anywhere except on the Rahib Islands.

4. Some of the inhabitants of the Rahib Islands have blue tattoo marks upon their bodies.

III. A certain club wishes to select the evening for its regular weekly meeting which would be most satisfactory to its members. Accordingly the secretary wrote to each member, asking what evening would be most satisfactory.

Can you suggest another question which would have been better for the secretary to ask?

IV. If a photographic plate be exposed to X-rays and then developed, black marks will be found upon it.

1. If upon developing a photographic plate you should find black marks upon it, what would you conclude?

2. Also if you should not find black marks upon it, what would you conclude?

V. If John agrees to join the football team provided Charles joins it, but Charles decided not to join it, what follows about John? If John joins, but Charles does not join, is John breaking his agreement?

The coefficients of correlation of the test results with the plane geometry-class records are computed for four of the groups. The other group was composed of teachers of secondary mathematics, and I have included the results on the test for the purpose of making a comparison of the questions with younger and with more mature students.

The formulæ used in this paper are as follows:

I. $$r = 2 \sin\left(\frac{\pi}{6}\rho\right), \quad \text{where} \quad \rho = 1 - \frac{6\Sigma D^2}{n(n^2-1)};$$

II. $$\text{P.E.} = 0.706\,\frac{1 - r^2}{\sqrt{n}}.$$

In these formulæ, r is the coefficient of correlation between the results of the test and the average of the class marks in plane geometry; D is the difference in ranks of the individual students in the test and in their plane geometry classes; n is the number of students in the group; and P.E. is the probable divergence of the true from the obtained coefficient of correlation.

The first group was composed of 29 girls in the Horace Mann School, Teachers College, New York. They were for the most part third-year high-school students, but their classification was by no means uniform as far as their mathematics was concerned, as the following analysis will show:

10 had studied plane geometry 1 year;
13 had studied plane geometry ½ year;
2 had studied plane geometry ¼ year;
2 had studied plane geometry ¼ year, and dropped it;
2 had studied plane geometry in other schools, and were credited.

In examining this group question II.—4 was inadvertently omitted from the list. Judging from the fact that this question proved for the other groups to be about average in difficulty, its omission was probably not material as far as the results are concerned.

The following table shows the percentage of the class scoring on each question:

TABLE I.

Question.	Percentage.
I.	67
II.—1	42
II.—2	73
II.—3	55
III.	17
IV.—1	7
IV.—2	83
V.	31

Table II. shows the percentage scoring the corresponding number of points:

TABLE II.

Points.	Percentage.
8	0
7	7
6	7
5	10
4	34
3	21
2	14
1	3½
0	3½

The correlation coefficient between these results and plane geometry marks, computed by Formula I., above, is $r = .25$, with a probable error of .12.

The low correlation and the comparatively high unreliability do not permit any reliable conclusion as to transfer. But as the members of this group show so little uniformity in point of time spent in plane geometry, as well as in other features of classification, the results are not surprising. As it is the P.E. is a little less than one half the coefficient of correlation.

However, as Professor Chambers has already pointed out, the questions were prepared not primarily to test the students but to determine the reliability of the questions as a measure of ability in general reasoning.

Referring to Table I., it will be seen that Questions IV.—1, III., and V. proved hardest for this group.

The second group was made up of 38 girls in the fourth-year

high-school grade of the Horace Mann School. Ten of these had plane geometry complete during the school year 1912–3, and the remaining twenty-eight during the school year 1913–4. I gave the test near the close of the first quarter of the school year 1914–5.

Table III. shows the percentage scoring on each question:

TABLE III.

Question.	Percentage.
I.	82
II.—1	57
II.—2	89
II.—3	55
II.—4	84
III.	42
IV.—1	34
IV.—2	87
V.	37

Table IV. shows the percentage scoring the corresponding number of points:

TABLE IV.

Points.	Percentage.
9	3
8	13
7	18
6	24
5	10
4	24
3	2½
2	5½
1	0

The coefficient of correlation and its unreliability for this group are as follows: $r = .75$, and P.E. $= .05$.

I attribute this high correlation and low unreliability in part to the opportunity the teachers had for accurately estimating the class standing of the students, having them as they did throughout the entire course in plane geometry; and to the frequency of similar tests in the Horace Mann School, thus tending to afford a normal and accurate measure of the class. Of course, in addition to these causes, there was unquestionably a transfer

in reasoning ability from some source. As to how much came from geometry and how much from other subjects can not be determined, except in so far as the test questions serve as a reliable measure of training given exclusively in geometry.

As in the first group, Table III. shows questions IV.—1, III., and V. to be the hardest for this group also, though not exactly in the same order.

The third group was composed of 84 girls in the third year of the Washington Irving High School, New York City. These girls began the study of plane geometry at the beginning of the second quarter of the school year 1913–4, and at the time of this test were at work in Book V. All of them had been together during their study of geometry, and probably longer, and in computing the correlation I used the average of the class marks for three quarters in plane geometry.

Table V. shows the percentage on each question:

TABLE V.

Question.	Percentage.
I.	58
II.—1	65
II.—2	93
II.—3	66
II.—4	59
III.	23
IV.—1	11
IV.—2	74
V.	17

Table VI. shows the percentage scoring the coresponding number of points:

TABLE VI.

Points.	Percentage.
7	7
6	19
5	36
4	15
3	14½
2	6
1	2½

The coefficient of correlation and its unreliability are as follows: $r = .31$, and P.E. $= .07$.

This group is probably a more nearly representative class for the high schools of our larger cities than any of the other groups.

Here again the results are similar to those of the two groups reported above with respect to the difficult questions.

The same questions, with the omission of II.—4, were given to a class of 24 young ladies and gentlemen in Teachers College. The title of the course is "Education 279," and the course is in the nature of a practicum in the teaching of mathematics.

Table VII. shows the percentage scoring on each question:

TABLE VII.

Question.	Percentage.
I.	83
II.—1	62
II.—2	83
II.—3	67
III.	67
IV.—1	83
IV.—2	83
V.	71

Table VIII. shows the percentage scoring the corresponding number of points:

TABLE VIII.

Points.	Percentage.
8	38
7	8
6	25
5	17
4	8
3	4

Questions IV.—1 and III., which were the most difficult for all the student groups, were not more difficult than the other questions for this adult group. This seemed to confirm my opinion, formed from the results of the first test, that question IV.—1 involved technical terms so unfamiliar to the average high-school student as to largely vitiate its value to test his reasoning ability; and that question III. involved the consideration of a problem which, while provided for in geometry, would not yield readily and naturally except as a result of experience in dealing

with similar situations. Accordingly, I prepared two questions of the same type as IV.—1 and III. I did not substitute these, but added them to the original list for the purpose of comparing results. I numbered these VI. and VII., and stated them as follows:

VI. If a man falls from the top of a three-story building bones of his body will be broken. Now, if you should find a man with broken bones lying on the sidewalk adjoining a three-story building, what would you conclude?

VII. A teacher wishes to change the hour of recitation of a certain class. He requests each member of the class to write and hand in on paper the most convenient hour. Could the student have given any better information in assisting the teacher to find a suitable hour? If so, what? (The teacher is free to meet the class at any hour.)

I gave the amended list to 38 young ladies and gentlemen in the East Tennessee State Normal School. These students would compare in classification approximately to the fourth-year high-school grade, being classified for most part as fourth-year academic in our school. The average age was a little over twenty. Practically all of them had completed plane geometry within a year. About one half of them had their geometry in our school, while the others were credited with it from other schools.

Table IX. shows the percentage scoring on each question:

TABLE IX.

Question.	Percentage.
I.	82
II.—1	84
II.—2	89
II.—3	76
II.—4	89
III.	29
IV.—1	11
IV.—2	79
V.	42
VI.	68
VII.	55

Table X. shows the percentage scoring the corresponding number of points:

TABLE X.

Points.	Percentage.
11	5
10	5
9	27
8	21
7	8
6	14
5	15
4	0
3	5
2	0
1	0

For the eighteen who had plane geometry in our school I computed the correlation of the test results with the class marks, which was as follows: $r = .52$, with P.E. $= .12$.

Comparing Table IX. with the corresponding tables for the other groups, I find that the questions which were most difficult for the other groups were also most difficult for this group so far as the original list is concerned. But it will be noted that while only 50 per cent. scored on IV.—1, 68 per cent. scored on VI., which is of the same type but involving more familiar terms; and that while 29 per cent. scored on III., 55 per cent. scored on VII., which is of the same type but involving a situation more in keeping with the experience of the students.

Professor Chambers suggested in his paper that some of his original questions could be modified so as to better serve the purpose for which he intended them. In keeping with that suggestion I am offering questions VI. and VII. as modifications of questions IV.—1 and III.

All of the groups tested so far, including the one reported by Professor Chambers, found question V. much more difficult than the average of the list, exclusive of questions IV.—1 and III. I believe, however, that this type of question should be included, as it involves an important principle in geometrical and general reasoning. Some of the students in "Education 279" and also in the State Normal School group made the point that the phrase "provided Charles joins" implies that John agrees to join "if and only if Charles joins." I do not find any authority for such interpretation, and I am inclined to the

opinion that the question could not be modified without largely impairing its value. Furthermore, I do not think the low scoring on this question is attributable to any appreciable extent to the wording.

I might add that I have followed Professor Chambers' method of scoring and computing in securing the results reported in this paper.

East Tennessee State Normal School,
Johnson City, Tenn.

NEW BOOKS.

In the Footprints of Napoleon. By JAMES MORGAN. New York: The Macmillan Company. Pp. 524. $2.50.

Before writing this story of the life of Napoleon Mr. Morgan started at his birthplace on the island of Corsica and traveled over the path of his career until his downfall and death. This made a journey of nearly twenty thousand miles. As a result we have a story of very great interest, giving us a more accurate concept of the real character of the man and a better understanding of the affairs in which he played such a prominent part. No matter what we have already read of Napoleon's life we will find new interest and profit in reading this.

The Evidence in the Case. By JAMES M. BECK with an Introduction by JOSEPH H. CHOATE. New York: G. P. Putnam's Sons. Pp. 275. $1.00 net.

Mr. Beck has given in this volume one of the clearest analyses which has as yet appeared of the causes of the war. His legal training has equipped him to take the official papers of all the warring countries and give us a judgment free from bias. After the war is over and its history comes to be written it will undoubtedly be found that this presentation of the case before the "Supreme Court of Civilization" will be correct. Thousands of readers will welcome this new and enlarged edition.

Our Dooryard Friends. By SARA V. PRUESSER. Chicago: The Platform. Pp. 204.

This is a very instructive series of sketches of a number of our common birds, and while they are common most people know little more about them than their names. Those who read it will find much of interest and will be more anxious to protect such good friends.

Schools of To-morrow. By JOHN DEWEY and EVELYN DEWEY. New York: E. P. Dutton and Company. Pp. 316. $1.00 net.

In this day of so many educational theories and fads it is refreshing to find a book which, while it may be more radical than some would like, is conservative enough to point the way towards which our best educational theory and practice is tending.

Professor Dewey points out what he believes are the fundamental principles of elementary education and indicates what our changing social conditions require. He illustrates his points by some of the successful experiments that are being tried at a few centers. Just how far this theory can be carried in secondary and higher education and just how it will be worked out remains for the future to develop.

A Budget of Paradoxes. By AUGUSTUS DE MORGAN. Second edition, edited by DAVID EUGENE SMITH. Chicago: The Open Court Publishing Co. Vol. I. Pp. 402. $3.50. Vol. II. Pp. 387. $3.50 net.

The editor and publishers of this work have placed present-day readers under a debt of gratitude for bringing out this rare work of delicious satire. The copious notes furnish the information necessary to its understanding at the present time, and the reader who takes it up will not wish to lay it down in a hurry.

Our Little Carthagenian Cousin of Long Ago. By CLARA VOSTROVSKY WINLOW. Boston: The Page Company. Pp. 127. 60 cents.

This story will reveal to the young mind interesting elements in the character of this remarkable people, and especially in the character of their great leader Hannibal. It is a worthy member of a splendid series.

Our Little Norman Cousins of Long Ago. By EVALEEN STEIN. Boston: The Page Company. Pp. 117. 60 cents.

The boy or girl who reads this book will have a very good knowledge of how the people of Normandy lived in the time of William the Conqueror. It is illustrated and well written.

The Complete Club Book. By CAROLINE FRENCH BENTON. Boston: The Page Company. Pp. 302. $1.25 net.

This is a guide book for the conduct of women's clubs, giving besides such instructions many topics of study. It also contains a model constitution and rules of order.

Analytic Geometry. By H. B. PHILLIPS. New York: John Wiley and Sons. Pp. 197.

This is intended as a brief course introductory to the calculus. Only the essentials are given, but the teacher and student who desire more can find additional material in the numerous examples. The first chapter takes up a review of the algebra used, and chapters nine, ten, and eleven take up three-dimensional geometry.

Diophantine Analysis. By R. D. CARMICHAEL. New York: John Wiley and Sons. Pp. 118. $1.25 net.

This introduction to Diophantine Analysis is No. 16 of the *Mathematical Monographs* series and satisfies a need which has been felt by many in not having a work which covered this ground. One will find in the text or exercises the results of essential importance which fall within the scope of an elementary work.

Carlyle, How to Know Him. By BLISS PERRY. Indianapolis: Bobbs-Merrill Company. Pp. 267. $1.25 net.

For one who has read Carlyle this book will be of great help in obtaining a more complete understanding of the man and his works. For one who has not as yet read him it will be an invaluable guide in reaching an intellectual appreciation of what Carlyle means to this country.

Browning, How to Know Him. By WILLIAM LYON PHELPS. Indianapolis: Bobbs-Merrill Company. Pp. 381. $1.25 net.

This should be a very welcome book for those who have tried without success to understand Browning. Professor Phelps has inspired thousands of young men with a love for the poetry of Browning and has furnished in this volume a regular guide book to the poet, having traveled to all the places Browning knew. All the author's poems are included as well as illuminating comments on them. With this as a guide no one need be unfamiliar with this modern poet. Other volumes of this series will be looked for with interest.

Anne of the Island. By L. M. MONTGOMERY. Boston: The Page Company. Pp. 326. $1.25 net.

Anne of the Island is a delightful sequel of *Anne of Green Gables* and *Anne of Avonlea,* and will be found as interesting as the other "Anne" books. In this we find the same Anne and share her pleasures and friendships through her four years of college at Redmond, and her vacations back at home at Green Gables with Marilla and the twins, one of them being as interesting as Anne herself; and at the last we have her romance.

The Theory of Relativity. By ROBERT D. CARMICHAEL. New York: John Wiley and Sons. Pp. 74. $1.00.

This is No. 12 of the series of *Mathematical Monographs* edited by Mansfield Merriman and Robert S. Woodward. The subject is one which has considerable interest though many physicists seem to pay little attention to it. For those who desire an introduction to the subject this book will prove very useful. Among others the following topics are considered: The postulates of relativity, the measurement of length and time, mass and energy, experimental verification of the theory.

How to Study and What to Study. By RICHARD L. SANDWICK. Boston: D. C. Heath & Co. Pp. 175. 60 cents.

Perhaps the most neglected phase of education and yet one of the most important is the formation of good mental habits. Teachers have not been taught how to study so they in turn do not teach others this art. It is of importance therefore that attention is drawn afresh to the subject by the appearance of this book which every teacher should read. His reasons why we should study mathematics may not be as complete as might be desired yet the book as a whole is very suggestive and will do a large amount of good.

Laird & Lee's Diary and Time-Saver maintains its high standard and as usual has something new. Besides the diary proper it contains the moon's phases for 1916, population of U. S. cities of 10,000 and up, Panama Canal statistics and map, calendars, wages table, surgical and medical hints, cash account pages, page for telephone numbers one uses, astrological signs for each day in the year, postage rates, etc. Attractive full flexible leather binding, gild edges, gold title, 25c. Press of Laird & Lee, Inc., Chicago.

School Algebra. By H. L. RIETZ, A. R. CRATHORNE, and E. H. TAYLOR. New York: Henry Holt & Company. First Course. Pp. xiii + 271. Pp. 235. Second Course.

The first course covers first-year algebra, including the usual topics. It is well written and well printed, and gives evidence of being teachable. The explanations are clear without being diffuse, and there is an abundance of exercises.

The second course covers intermediate algebra, including a review of the most important parts of the first year course and the usual topics of the second year. It also contains excellent material, especially in the lists of problems. While the problems are interesting and of considerable variety, they do not presuppose an undue knowledge of the subjects. There are several features worthy of comment, including a restrained use of history, the use of determinants in solving linear equations, and the introduction of functionality. The books are worthy of careful examination.

A Handbook of the best Private Schools of the United States and Canada. By PORTER E. SARGENT. Boston.

This book contains lists of boys' schools, girls' schools, co-educational schools, special schools, and summer camps. It also arranges the schools and certain of their characteristics in tabular form for convenient reference and comparison.

In addition to the school lists, it gives directories of associations, educational publications, publishers, teachers' agencies, and dealers in school supplies and equipment of various kinds.

The book is well indexed, and should prove an invaluable reference book to the busy executive or teacher. It is neither complete nor entirely accurate as yet, but the field is large, and succeeding years should see great improvement in this respect.

NOTES AND NEWS.

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New subscribers who send $2.00 for 1916 will receive free a copy of The Companion Home Calendar for 1916, in addition to all the remaining 1915 issues from the time the subscription is received.—*The Youth's Companion,* Boston, Mass.

Peace Prize Contest, under the auspices of The American School Peace League. Open to pupils of all countries. Two sets of prizes, to be known as the Seabury Prizes, are offered for the best essays on one of the following subjects:

1. The Opportunity and Duty of the Schools in the International Peace Movement. Open to Seniors in the Normal Schools.

2. The Influence of the United States in Advancing the Cause of International Peace. Open to Seniors in the Secondary Schools.

Three prizes of seventy-five, fifty and twenty-five dollars will be given for the best essays in both sets.

Contest closes March 1, 1916. Conditions of the Contest: Essays must not exceed 5,000 words (a length of 3,000 words is suggested as desirable), and must be written, preferably in typewriting, on one side only of paper, 8×10 inches, with a margin of at least $1\frac{1}{4}$ inches. Manuscripts not easily legible will not be considered.

The name of the writer must not appear on the essay, which should be accompanied by a letter giving the writer's name, school, and home address, and sent to Mrs. Fannie Fern Andrews, Secretary, American School Peace League, 405 Marlborough Street, Boston, Mass., not later than March 1, 1916. Essays should be mailed flat (not rolled).

The award of the prizes will be made at the Annual Meeting of the League in July, 1916.

Information concerning literature or the subject may be obtained from the Secretary.

THE United States Bureau of Education has just issued Bulletin 1915, No. 35, on "Mathematics in the Lower and Middle Commercial and Industrial Schools of Various Countries Represented in the International Commission on the Teaching of Mathematics."

This bulletin has been prepared by Dr. E. H. Taylor, with the editorial co-operation of the members of the Commission in the United States. It is a bulletin of 96 pages, and will be furnished to teachers of mathematics upon application to the United States Bureau of Education at Washington.

The United States Bureau of Education at Washington has recently issued Bulletin 1915, No. 39, by Dr. I. L. Kandel, on "The Training of Elementary School Teachers in Mathematics" in the countries represented in The International Commission on the Teaching of Mathematics. This is issued with the co-operation and under the direction of the American members of the International Commission on the Teaching of Mathematics, Professors D. E. Smith, W. F. Osgood, and J. W. A. Young.

This bulletin shows the excellent preparation of teachers of elementary mathematics in several of the leading countries in contrast to the meager academic preparation in certain other

countries. It is sure to be of interest and value to all who have to do with elementary education in mathematics in the United States.

THE International Commission on the Teaching of Mathematics has recently issued, through the Bureau of Education at Washington, an important bulletin on the teaching of elementary and secondary mathematics in all the leading countries of the world. This bulletin, No. 45, 1914, was prepared by J. C. Brown, and sets forth the nature of the mathematics taught in every school year, from the first through the twelfth, in the standard type of school. It should be in the hands of every teacher of mathematics, and may be secured by addressing the United States Commissioner of Education, Washintgon, D. C.

THE New York Section of The Association of Teachers of Mathematics in the Middle States and Maryland held its twenty-fourth regular meeting at Hunter College, Friday, November 12, 1915. This was a joint session with the Mathematics Section of the New York High School Teachers Associa-

A Review of High-School Mathematics

By WILLIAM D. REEVE and RALEIGH SCHORLING

Head of the Department of Mathematics in the University of Minnesota High School — Instructor in Mathematics in the University of Chicago High School

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Some Opinions

"The book has impressed me so favorably that I shall recommend it to our teachers of mathematics review."—J. REMSON BISHOP, Principal, Eastern High School, Detroit, Michigan.

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"Well worked out and meets a real need in high-school mathematics. It is very valuable in all review work."—E. C. UNVERFERTH, Tulsa High School, Tulsa, Oklahoma.

x + 80 pages, 12mo, cloth; $0.40, postage extra (weight 10 oz.).

The University of Chicago Press, - - Chicago, Illinois

tion and the following program made an interesting meeting: "Arithmetic in the High Schools," by Prof. David Eugene Smith, Teachers College, Columbia University; "Mathematics Relays" with pupil demonstration, by Mr. T. H. McCormick, High School of Commerce; "Arithmetic in the High Schools," Round Table Discussion. Leaders: Mr. W. H. Schlauch, H. S. Commerce; Dr. L. W. Goldrich, Principal P. S. 62; Mr. W. E. Grady, Principal P. S. 64; Mr. F. J. Brownscombe, National Cloak and Suit Company; Mr. Wm. Fox Roantree, Teachers Training School.

Do you realize the fact that *St. Nicholas* is forty-two years old? You read it perhaps when you were young. Are your children or your nephews and nieces having it to-day?

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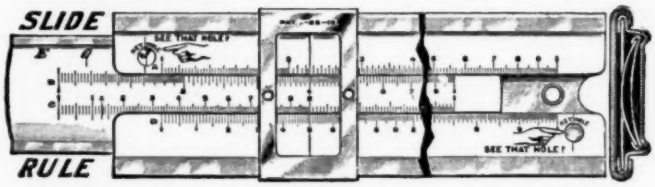

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ful biography of him. He has now rewritten that biography for children, and "The Boys' Life of Mark Twain" will be a feature of *St. Nicholas* in 1916.

HOWEVER much people wish it would stop, yet the war, while it lasts, is the most gigantically interesting event of modern times. Forthcoming numbers of *The Century* will have contributions on every aspect of it from such authoritative writers as Hilaire Belloc, W. Morgan Shuster, Arthur Bullard, J. D. Whelpley, T. L. Stoddard, Eric Fisher Woods, Herbert Adams Gibbons, E. D. Schoonmaker, Estelle Loomis. Essays with a pulse. That is to say, men and women will write for you as brilliantly, as vividly, as entertainingly as if they were telling you a story—only they will be telling you the truth. Coming contributions from G. K. Chesterton, Simeon Strunsky, E. Björkman, Amy Lowell, Eugene Wood, Brian Hooker, Harvey O'Higgins, William Caine, H. Addington Bruce, C. F. Carter, Randolph S. Bourne, S. P. Orth.

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The High School Teachers Association of California has recently recommended the reading of a first class journal in order to broaden the scholarship of the secondary teachers of mathematics, and the Monthly was unanimously named for this purpose.

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